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(r*g*)* CONNECTEDNESS AND (r*g*)* COMPACTNESS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce the concept of (r*g*)*connectedness and (r*g*)* compactness and study some of their properties

KEYWORDS: (r*g*)* Closed Sets, (r*g*)* Open Sets, (r*g*)* Open Cover, (r*g*)* Irresolute Maps, (r*g*)* Continuous Maps

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1. INTRODUCTION

N Levin [4] introduced the concept of generalized closed set in topological spaces. In 1974, Das defined the concept of semi-connectedness in topology and investigated its properties. Balachandran, Sundaram and Maki [1] introduced a class of compact space called Go-compact space and Go-connected space using g-open cover. Easwaran and pushpalatha [3] introduced and studied $\mathfrak{I}$*-generalized compact spaces and $\mathfrak{I}$* connected spaces A. M. Al. Shibari [9] S. S. Benchalli and Priyanka M. Bansali introduced rg-compact spaces and rg-connected spaces and study some of their properties**.** In this paper we introduce the concept of (r*g*)*connectedness and (r*g*)* compactness and study some of their properties.

Definition 2.1: A subset A of a Topological Space is said to be a (r*g*)*closed set [6] if cl(A) $\subset$ U whenever A $\subseteq$ U and U is r*g*- open. The complement of (r*g*)*closed set is (r*g*)*open.

Definition 2.2: A map f: (X, τ) $\rightarrow$ (Y, σ) is called (r*g*)*-continuous [7] if the inverse Image of every closed set in (Y, σ) is (r*g*)*-closed in (X, τ).

Definition 2.3: A map f: (X, τ) $\rightarrow$ (Y, σ) is said to be a (r*g*)*-irresolute map [7] if $f^{-1}(V)$ is a (r*g*)*-closed set in (X, τ) for every (r*g*)*-closed set V of (Y, σ).

Definition 2.4: A Space (X, τ) is called (r*g*)*$T_{1/2}$ space [8] if every (r*g*)* closed set in it is closed.

Definition 2.5: Let X be Topological space. Let A be a subset of X. (r*g*)* closure [8] of A is defined as the intersection of all (r*g*)* closed sets containing A.

Definition 2.6: A property P holding good for a topological space (X, $\mathfrak{I}$) and which is also Hold good for every subspace of the topological space is called Hereditary property.

Definition 2.7: A collection C̀ of subsets of X is said to have the finite intersection property (FIP) if for every finite sub collection {C_1, C2, C_n} of C̀, the intersection $C_1 \cap C_2 \cap . . . . \cap C_n$ is non empty.

3. (r*g*)* CONNECTEDNESS

Definition 3.1: A Topological space X is called (r*g*)* connected if X cannot be written as a Union of two non-empty disjoint (r*g*)* open sets.

Definition 3.2: A subset A of X is (r*g*)* connected if it is (r*g*)* connected as subspace of X.

Example 3.3: Let X = {a,b,c), τ = {φ, X, {a},{b}, {a,b}}. (r*g*)* closed sets are φ X, {c} {b,c},{a,c} . (r*g*)* open sets are φ, X,{a},{b},{a,b}. Here X cannot be written as the Union of two non empty, disjoint (r*g*)* open sets .Hence X is (r*g*)* connected.

Another way of defining (r*g*)* connectedness is as follows.

Definition 3.4: A space X is (r*g*)* connected iff the only subsets of X that are both (r*g*)* open and

(r*g*)*- closed in X are the empty set and X itself.

Proof: If A is a non-empty proper subset of X that is both (r*g*)* open and (r*g*)* closed in X, then the sets A and X - A constitute a separation of X, Since they are (r*g*)* open, disjoint and non- empty and their Union is X .Conversely if X and Φ are the only (r*g*)* closed subsets TPT that X is (r*g*)* connected. If not Let X=A ∪ B Where A and B are two non empty, disjoint (r*g*)* open sets which ⇒ B= X-A which is both (r*g*)*open and (r*g*)* closed which is a contradiction. Therefore X is connected.

Example 3.5: Let X = {a, b}, ℑ = {φ, X, {a}} (r*g*)* Closed sets are φ X, {b}. (r*g*)* Open sets are φ X, {a}. The only subsets of X that are both (r*g*)* open and (r*g*)* closed are φ and X. Hence X is (r*g*)*connected.

Remark 3.6: Any indiscrete space with two points is (r*g*)*connected. A two point set with discrete topology is not (r*g*)* connected.

Example 3.7: Let X = {a, b}, ℑ = {φ, X, {a}, {b}}, ℑ Closed = {φ, X, {a}, {b}} (r*g*)* Open sets are are φ, X, {a}, {b}. Here X is not (r*g*)*connected.

Example 3.8: Let X be the subspace [-1, 1] (with order topology) of the Real line. Consider the sets [-1, 0]. Since every closed set is (r*g*)* closed, [-1, 0] is (r*g*)* closed and**]** 0, 1] is (r*g*)* open. Then they are disjoint and non-empty and their union is X = [-1, 1] but] 0, 1] is not (r*g*)* open. Hence There is no separation and hence X is (r*g*)* connected.

Remark 3.9: The (r*g*)* connectedness property is not a hereditary property. The following example proves this.

Example 3.10: Let X={a, b ,c}, ℑ={$\emptyset$,X,{a},{b},{a, b}}

(r*g*)* open sets= {$\emptyset$,X,{a},{b},{a, b}}

(r*g*)* closed sets= {$\emptyset$,X,{c},{a, c},{b, c}}

Here the only subsets which are both (r*g*)* open and (r*g*)* closed sets are $\emptyset$ and X and hence X is (r*g*)* connected. Let Y= {a, b}

The relative topology ℑ*= {A ∩ Y/A ∈ ℑ}

ℑ*= {Y, ∅,{a},{b}}

(r*g*)* open sets= {∅,Y,{a},{b}}

Now {a} is (r*g*)* open. Its complement {b} is also (r*g*)* open.

{a} is both (r*g*)* open and (r*g*)* closed set.

⇒ Y is not (r*g*)* connected.

Theorem 3.10: Every (r*g*)* connected space is connected.

Proof: Let X be a (r*g*)* connected space. If possible Let X be not connected.

Then X can be written as X = A ∪ B Where A and B are disjoint, nonempty open sets. But Every open set is (r*g*)* open set in X ⇒ X is not (r*g*)* connected which is a contradiction. Hence X is connected.

The Converse of the above theorem is true whenever X is (r*g*)* $T_{1/2}$.

Theorem 3.11: Let X be (r*g*)* $T_{1/2}$. Every connected space is (r*g*)* connected.

Proof: Let X be connected. To Prove that: X is (r*g*)* connected. Suppose X is not (r*g*)* connected. Let A & B be any two (r*g*)* open subsets of X such that X = A ∪ B such that A ∩ B = φ. Since X is (r*g*)*$T_{1/2}$, every (r*g *)* open set is open and hence A & B are open sets of X.

Which contradicts that X is connected ∴ X is (r*g*)* connected.

Theorem 3.12: Let X be a Topological space. Let Y be (r*g*)* connected subspace of X. If X can be written as the Union of two (r*g*)* open sets of X then Y lies entirely within A or B.

Proof: Let X=A ∪ B and A ∩ B= φ. Since A and B are (r*g*)* open in X then A ∩ Y and B ∩ Y are (r*g*)* open in Y. Now A ∩ Y and B ∩ Y are disjoint and their union is Y. If both were nonempty then they form a separation of Y which is a contradiction. Hence one of them is empty. Suppose A ∩ Y = φ Then Y = (A ∩ Y) ∪ (B∩Y) => Y = (A ∩B) ∪ Y= φ ∪ (B∩Y) => Y ⊂ B. similarly we can discuss the case B ∩ Y = φ.

Theorem 3.13: The (r*g*)* closure of (r*g*)* connected set is (r*g*)* connected.

Proof: Let E be a (r*g*)* connected subset of (X, ℑ). TST (r*g*)* cl(E) is connected. If not (r*g*)* cl(E) can be written as (r*g*)* cl(E) = A ∪ B where A,B are -disjoint (r*g*)* open sets . Now E ⊂ (r*g*)*cl(E) = A ∪ B ⇒ E ⊂ A or E ⊂ B . E ⊂ A ⇒(r*g*)*cl E ⊂(r*g*)* clA .⇒ (r*g*)*cl E ∩ B ⊂ (r*g*)*cl A ∩ B =φ

⇒(r*g*)*cl E ∩ B = φ ------ (1)

Also(r*g*)* cl E =A ∪ B ⇒ B ⊂ (r*g*)*clE implies B ∩ (r*g*)*cl E = B -------- (2)

From (1) & (2), B =φ which is a contradiction. Similarly if E ⊂ B we can get A = φ.

∴ (r*g*)*cl(E) must be (r*g*)* connected.

Theorem 3.14: If E is a subset of a Topological space (X, ℑ) then (r*g*)* closure of E is, (r*g*)* connected iff E is not the Union of any two non-empty sets A and B such that (r*g*)*cl A ∩ (r*g*)*cl B = φ

Proof: Let (r*g*)*cl E be (r*g*)* connected. Suppose E is the Union of two non-empty (r*g*)* open sets A and B such that (r*g*)* cl(A) ∩ (r*g*)* cl(B)= φ.

Now E= A ∪ B⇒ (r*g*)*cl (E) = (r*g*)* cl (A ∪ B) = (r*g*)*cl(A) ∪ (r*g*)* cl (B)

Also since (r*g*)* cl((r*g*)*cl (A)) = (r*g*)* cl (A).

(r*g*)* cl (A) ∩ (r*g*)* cl (B) = φ ⇒ (r*g*)* cl {(r*g*)*cl(A)} ∩ (r*g*)*cl (B) = φ --------- (1)

Similarly we can get, (r*g*)* cl (A) ∩ (r*g*)* cl ((r*g*)*cl(B))= φ --------- (2)

From (1) & (2), we can conclude that (r*g*)*cl E has a separation which implies (r*g*)*cl E is disconnected, which is a contradiction

∴ E cannot be expressed as a Union of two non-empty disjoint (r*g*)* open sets such that

(r*g*)* cl (A) ∩ (r*g*)*cl (B) = φ.

Conversely

If E is not the union of two non-empty disjoint (r*g*)* open sets such that (r*g*)* cl (A) ∩ (r*g*)* cl (B) = φ.

To prove (r*g*)*cl(E) is (r*g*)*connected. If not E ⊂ (r*g*)*cl(E)= A ∪B where A,B are -disjoint (r*g*)* open sets which is a contradiction . Hence (r*g*)*cl(E) is connected.

Theorem 3.15: The Union of any family of (r*g*)* connected sets having non-empty intersection property is (r*g*)* connected.

Proof: Let {E_α: α϶Λ} be a family of (r*g*)* connected subsets with the property that ∩ {E_α: α϶Λ} is non – empty. Let E= ∪ {E_α: α϶Λ}. To prove that E is (r*g*)* connected. If not, E can be written as Union of two non-empty disjoint (r*g*)* open sets such that ∪ E_α=E=A ∪ B and E_α ⊂ A ∪ B, for every α.

Since each E_α is connected, E_α ⊂ A or E_α ⊂ B for each α϶Λ⇒ ∪ E_α ⊂ A or ∪ E_α ⊂ B

⇒ E ⊂ A or E ⊂ B- - - - - (1)

Since ∩ {E_α : α϶Λ} is non-empty, Let x Є ∩ {E_α : α϶Λ}.Then xЄE_α, for every α϶Λ

Hence xЄE = ∪{ $E_{\alpha:}$α϶Λ} ∴ xЄE ⇒ xЄA or xЄB [by(1)]. x cannot belong to both A and B.

∴ if xЄA then x∉B⇒ E⊄B (by 1)

⇒ E ⊂ A, which is a contradiction. ⇒ E must be (r*g*)* connected.

Theorem 3.16: The Union of any family of (r*g*)* connected subsets with the property that, one of the member of the family intersects every other member is a (r*g*)* connected set

Proof: Let { E_α : α ϵΛ } be a family of (r*g*)* connected subsets of (X, ℑ) with the property that one of the members say $E\alpha_0$ intersects every other member. (i.e.)$E\ \alpha_0 \cap E\alpha \neq \varphi$, for every αϵΛ

To Prove: E= ⋃ E_α is connected. Now $E\ \alpha_0 \cup E\alpha$ being the union of (r*g*)* connected subset having non-empty intersection is a (r*g*)* connected set.

Now, let $E\alpha_p$ and $E\alpha_q$ be any two members of the family so that $E\alpha_0 \cap E\alpha_p \neq \varphi$ and $E\alpha_0 \cap E\ \alpha_q \neq \varphi$

Now $(E\ \alpha_0 \cup E\ \alpha_p) \cap (E\alpha_0 \cup E\alpha_q) = E\alpha_0 \cup (E\alpha_p \cap E\alpha_q) \neq \varphi$ -------(2)

∴ $\cap (E\alpha_0 \cup E\alpha) = E\alpha_0 \cup (\cap E\alpha) \neq \varphi$ [since $E\alpha_0 \neq \varphi$]

⇒ $\cup (E\alpha_0 \cup E\alpha)$ is connected.

⇒ $E\alpha_0 \cup (\cup E\alpha) = \cup E\alpha$ is connected.

Since it is given that $E\alpha_0$ intersects every other member of the family and $E\alpha_0 \neq \varphi$,

We conclude that $(E\alpha_0 \cup E\alpha_p) \cap (E\alpha_0 \cup E\ \alpha_q) \neq \varphi$, $p \neq q$------------------ (3)

⇒The collection $\{E\alpha_0 \cup E\alpha : \alpha\epsilon\Lambda\}$ has non-empty intersection. ∴ It is (r*g*)* Connected.

(i.e) $\cup\{E\alpha_0 \cup E\alpha : \alpha \in \Lambda\}$ is a (r*g*)* connected set or E= $\cup\{E\alpha / \alpha \in \Lambda\}$ is a (r*g*)* connected

Theorem 3.17: Let A be a (r*g*)* connected subspace of X. If A ⊂ B ⊂ (r*g*)* cl (A) then B is also (r*g*)* connected.

Proof:

Let A be (r*g*)* connected.

To Prove that: B is (r*g*)* connected.

If, not let B = C ⋃ D Since A ⊂ B, A must lie entirely in C or D.

Suppose A ⊂ C then (r*g*)* cl (A) ⊂ (r*g*)* cl (C)

(r*g*)* cl (A) ⋂ D ⊂ (r*g*)* cl (C) ⋂ D = φ

Now D ⊂ B, And B ⊂ (r*g*)* cl (A) .Now φ ⊂ D=D⋂B ⊂ (r*g*)* cl (A)⋂ D ⊂ φ

⇒ D = φ which is a contradiction. ∴ B is (r*g*)* connected.

Remark 3.18: In the above theorem if we replace B by (r*g*) cl (A) we can get Theorem 3.13

Theorem 3.19: Let f :X → Y be (r*g*)* continuous and onto. If X is (r*g*)* connected, Then Y is also connected.

Proof:

Suppose Y is not connected. Let Y = A ⋃ B where Aand B are disjoint non-empty open sets in Y.

Since f is (r*g*)* continuous, f^{-1} (A) and f^{-1} (B) are disjoint non-empty (r*g*)* open sets.

Since f is onto, f(X) = Y. ∴ We have X = f^{-1} (A) ∪ f^{-1} (B)

Which contradicts the fact that X is (r*g*)* connected. ∴ Y is connected.

Theorem 3.20: If f : X → Y is a (r*g*)* irresolute and onto, X is (r*g*)* connected then Y is (r*g*)* connected.

Proof:

Suppose Y is not (r*g*)* connected then Y = A ∪ B Where A & B are non-empty disjoint (r*g*)* open sets. Since f is (r*g*)* -irresolute, f^{-1} (A) and f^{-1} (B) are (r*g*)* open sets and X = f^{-1} (A) ∪ f^{-1} (B) ⇒ X is not (r*g*)* connected which is a contradiction. ∴ Y is (r*g*)* connected.

4. (r*g*)* compact space.

Definition 4.1: A collection $\{G_\alpha : \alpha \in \Lambda\}$ of (r*g*)* open sets in a topological space X is called a (r*g*)* Open cover of a subset A of X, If $A \subset \cup \{G_\alpha : \alpha \in \Lambda\}$.

Definition 4.2: A Topological space X is (r*g*)* compact if every (r*g*)* open cover has a finite subcover.

In other words if $\mathcal{C} = \{G_\alpha : \alpha \in \Lambda\}$ of (r*g*)*opensets then $\mathcal{C}$ is a (r*g*)*open cover for X iff $X = \cup \{G_\alpha : \alpha \in \Lambda\}$. Now If there exists $G\alpha_1\ G\alpha_2\ G\alpha_3 \ldots \ldots G\alpha_n$ in this collection $\mathcal{C}$ such that $X = \cup \{G_{\alpha i} : i=1,2. . .n\}$ then X is said to be (r*g*)* compact space.

Remark 4.3: Consider the discrete Topological space.

Case: 1 Let X be finite.

Then the number of (r*g*)* open subsets of X is also finite so every (r*g*)* covering of X is finite and hence any (r*g*)* sub cover of X is also finite so that it is (r*g*)* compact. In particular every finite subset of a Topological space is always (r*g*)* compact.

Case: 2 Let X be infinite

Now C = {{x}/x ϵ X} is a infinite (r*g*)* open covering for X as X =∪ {{x}/x ϵ x} and hence there does not exists any finite sub collection C’ such that X is the Union of that collection. Hence it does not have a finite sub cover. Thus an infinite discrete topological space is not (r*g*)* compact. In particular every infinite subset of a discrete topological Space is not (r*g*)* compact.

On the other hand if we consider the indiscrete topological space then C = {X} such that X = ∪ {X}, then C is a (r*g*)* Covering for X which consists of only one set and hence finite. Therefore X is a (r*g*)* compact space.

Remark 4.3: By Remark 1 Every finite set is (r*g*)* compact

Example 4.4: Let X = {a, b, c} $\mathfrak{I}$ = { φ, X, {a}, {b}, { a, b }}. (X, $\mathfrak{I}$) is (r*g*)* compact Since X is finite.

Definition 4.5: A subset B of a Topological space is said to be (r*g*)* compact if B is (r*g*)* compact as a subspace of X.

Theorem 4.6: If a map f: X → Y be (r*g*)* irresolute and a subset B of X be (r*g*)* compact relative to X, then f (B) is (r*g*)* compact relative to Y.

Proof: Let { A_α : α∈Λ } be any collection of (r*g*)* open subsets of Y such that

f (B)⊂∪ { A_α : α∈Λ } then B ⊂ ∪ {f^{-1} (A_α) : α∈Λ } but B is (r*g*)* compact relative to X .

Therefore, There exists a finite sub cover $B \subset \bigcup_{i=1}^{n} \{f^{-1}(A_i)\} \Rightarrow f(B) \subset \bigcup_{i=1}^{n} \{(A_i)\}$

⇒ f (B) is (r*g*)*compact.

Hence we can prove the following:

Theorem 4.7: A (r*g*)* continuous image of a (r*g*)* compact space is compact.

Theorem 4.8: If f: X→ Y is (r*g*)* irresolute and bijection and X is (r*g*)* compact then Y is a (r*g*)*compact space.

Proof: Let { A_α : α∈Λ } be a (r*g*)* open cover of Y .Then Y = ∪ A_α . Since f is a bijection, we have f (X) = Y or f^{-1} (Y) = X But f^{-1} (∪ A_α) = ∪ {f^{-1} (A_α)} Since f is (r*g*)* irresolute f^{-1} (A_α) is (r*g*)* open for each α∈Λ . But X is (r*g*)* compact therefore there exists finitely many indices $\alpha_1, \alpha_2. . . . \alpha_n$ such that

X= ∪ f^{-1} ($A_{\alpha i}$). ∴ Y=f(X)=f(∪ f^{-1} ($A_{\alpha i}$)) = ∪ $A_{\alpha i}$ Where i=1,2,. . .n.

⇒ Y is (r*g*)* compact.

Theorem 4.9: Every (r*g*)* closed subset of a (r*g*)* compact space is (r*g*)* compact relative to X .

Proof: Let A be a (r*g*)* closed subset of X. Then A^c is (r*g*)* Open. Let C= {G_α: α ϵ Λ} be a (r*g*)* open cover of A (by subsets of X) Then Let M = C ∪ A^c is an (r*g*)* open cover of X. That is X ⊂∪ {G_α : αϵΛ} ∪ A^c

Since X is (r*g*)* compact M has a finite sub cover say (G_1, ∪ G_2 . . . ∪ G_n) ∪ A^c

But A and A^c are disjoint hence A ⊂ G_1 ∪ G_2 . . . ∪ G_n . The open cover C has a finite (r*g*)* sub cover . Hence A is (r*g*)* Compact.

The following example shows that a (r*g*)* compact subset of a Topological Space need be not (r*g*)* closed.

Example 4.10: Let X = {a, b, c, d}, ℑ = { φ, X, {a},{b},{a,b},{a,b,c}}

(r*g*)* Open sets are Φ, X, {a, b, c}, {b, c},{a, c} {a, b}, {c},{b}, {a}

Let A = {a, b, c },then A can be written as, A = {a, c} ∪ {a, b} ∪ {b}

A is (r*g*)* compact. But A is not (r*g*)* closed.

Theorem 4.11: A space X is (r*g*)* compact iff each family of (r*g*)* closed subsets of X with the finite intersection property has non empty intersection.

Proof: Let X be (r*g*)*compact . Let A be any collection of (r*g*)* closed sets with F . I . P

Let $\mathcal{A}$ = {F_α : αϵΛ } be an arbitrary collection of (r*g*)* closed subsets of X with F.I.P

So that $\cap$ {$F_{\alpha i}$: i=1,2. . .n : } $\neq \varphi$ . . . . (1)

Now TPT: $\cap$ { F_α : $\alpha\epsilon\Lambda$ } $\neq \varphi$

Suppose this is not true. Then we have $\cap$ { F_α : $\alpha\epsilon\Lambda$ } = φ.

By taking complement we have $\cup$ {$F\alpha^c$: $\alpha\epsilon\Lambda$ } = X

But each F_α is (r*g*)* closed, $F\alpha^c$ is (r*g*)* open

∴ {F_α^c : $\alpha\epsilon\Lambda$ } becomes an (r*g*)* open cover for X. But X is (r*g*)* compact.

∴ There exists finitely many indices $\alpha_1, \alpha_2. . . . \alpha_n$ such that X= $\cup$ {$F_{\alpha i}^c$: i=1,2,. . . n}

Taking complement both sides We get φ = $\cap$ {$F_{\alpha i}$: i=1,2,. . . n} which is a contradiction to (1).

Hence $\cap$ {F_α : $\alpha\epsilon\Lambda$ } $\neq \varphi$

Conversely suppose any collection of (r*g*)* closed sets with FIP has a non empty

Intersection Let $\mathcal{B}$ = {G_α : $\alpha\epsilon\Lambda$ } where B is a (r*g*)* open cover of X and hence

X = $\cup$ {G_α : $\alpha\epsilon\Lambda$} Taking complements we have φ = $\cap$ {G_α : $\alpha\epsilon\Lambda$]

But G_α^c is (r*g*)* closed. ∴ The collection of (r*g*)* closed subsets has empty intersection.

∴ It does not satisfy F.I.P. Hence there exists a finite number of (r*g*)* closed sets

$G_{\alpha i}^c$ where i = 1,2,. . . n with empty intersection. That is { $G_{\alpha i}^c$: i = 1,2,. . . n } = φ

Again taking complement { $G_{\alpha i}^c$: i = 1,2,. . . n } = X

∴ B Has a finite subcover. Hence X is (r*g*)* compact.

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